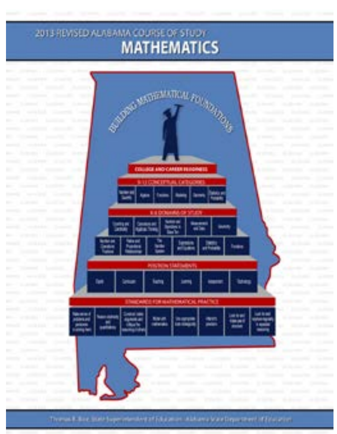
Alabama
College- and Career-Ready
Standards & Support
Navigating Success for All...

6 - 12 Math

Facilitator Notes



September 2014

Anticipation Guide

Suggestions for discussing Anticipation Guide.

Overview:

1. In Grades 6 – 8, students build on the important concept of whole numbers and fractions as points on the number line in order to understand the rational numbers as a number system.
(+ Agree, but I would add that students build on a second important concept.)(Overview, Page 2) In Grades 6–8, students build on two important conceptions which have developed throughout K–5, in order to understand the rational numbers as a number system. The first is the representation of whole numbers and fractions as points on the number line, and the second is a firm understanding of the properties of operations on whole numbers and fractions.

2. In order to understand the addition of numbers symbolically, students should represent sums as concatenated lengths on the number line.
(O, Disagree, because representing sums as concatenated lengths is another way of thinking of sums different from symbols.)(Last paragraph, Page 2) Representing sums as concatenated lengths on the number line is important because it gives students a way to think about addition that makes sense independently of how numbers are represented symbolically.

6th Grade:

3. Understanding direction on the number line is critical for 6th grade students. **(+ Agree, Directed measurement scales for temperature and elevation provide a basis for understanding positive and negative numbers as having a positive or negative direction on the number line.)(Paragraph 3, Page 7)**

4. Comparing negative numbers requires closer attention to the relative positions of the numbers on the number line rather than their magnitudes**. (+ Agree, With the introduction of negative numbers, students gain a new sense of ordering on the number line………—comparing negative numbers requires closer attention to the relative positions of the numbers on the number line rather than their magnitudes.)(1st whole paragraph, Page 8)**

7th Grade: (Only add, sub., mult and divide) Number????

5. With the introduction of direction on the number line, there is a distinction between the distance from *a* and *b* and how you get from *a* to *b*. **(+ Agree, With the introduction of direction on the number line, there is a distinction between the distance from *a* and *b* and how you get from *a* to *b*. The distance from -3 to -5 is 2 units, but the instructions how to get from -3 to -5 are "go two units to the left." The distance is a positive number, 2, whereas "how to get there" is a negative number -2.)(Paragraph 2, Page 10)**

6. The absolute value of *p* – *q* is just the distance from *p* to *q*. **(+ Agree, but adding "regardless of direction" makes the sentence stronger. In Grade 6 we introduce the idea of absolute value to talk about the size of a number, regardless of its sign. It is always a positive number or zero. If *p* is positive, then its absolute value l*p*l is just *p*; if *p* is negative then l*p*l = -*p*. With this interpretation we can say that the absolute value of *p* – *q* is just the distance from *p* to *q*, regardless of direction.)(Last sentence in paragraph 2, Page 10)**

Grade 8:

7. Every fraction can be represented as a finite decimal. **(O Disagree, 1st and 2nd paragraph. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.)**

8. Irrational numbers correspond to a point on the number line. **(+ Agree, Last paragraph, Page 14, The possibility of infinite repeating decimals raises the possibility of infinite decimals that do not ever repeat. From the point of view of the decimal address system, there is no reason why such number should not correspond to a point on the number line.)**

High School:

9. Rational exponents preserve the laws of exponents. **(+ Agree, 1st paragraph, Page 17, Because rational exponents have been introduced in such a way as to preserve the laws of exponents, students can now use those laws in a wider variety of situations.)**

10. In applications of mathematics, the distinction between rational and irrational numbers is irrelevant. **(+ Agree, Last paragraph Page 17. Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with finite decimal approximations (and therefore with rational numbers), thinking about the properties of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments and attending to precisions (MP.3, MP.6).**

Illustrative Mathematics

6.NS Comparing Temperatures

Alignments to Content Standards

- Alignment: 6.NS.C.7.b

Tags

- *This task is not yet tagged.*

a. Here are the low temperatures (in Celsius) for one week in Juneau, Alaska:

Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
5	-1	-6	-2	3	7	0

Arrange them in order from coldest to warmest temperature.

b. On a winter day, the low temperature in Anchorage was 23 degrees below zero (in $^\circ C$) and the low temperature in Minneapolis was 14 degrees below zero (in $^\circ C$). Sophia wrote,

> *Minneapolis was colder because* $-14 < -23$.

Is Sophia correct? Explain your answer.

c. The lowest temperature ever recorded on earth was $-89^\circ C$ in Antarctica. The average temperature on Mars is about $-55^\circ C$. Which is warmer, the coldest temperature on earth or the average temperature on Mars? Write an inequality to support your answer.

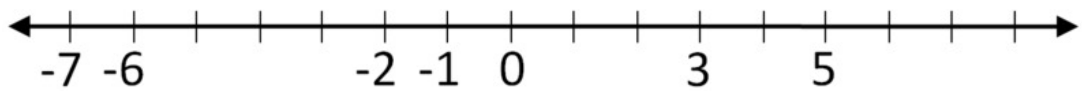
Commentary

The purpose of the task is for students to compare signed numbers in a real-world context. It could be used for either assessment or instruction if the teacher were to use it to generate classroom discussion.

Solutions

Solution: Solution

a. Let’s begin by plotting them all on the same number line.



The number line has positive numbers to the right of zero and negative numbers to the left of zero. This means that numbers father to the right are always greater than those to the left. In terms of temperature, the coldest temperature (the least number) is all the way to the left, and the warmest temperature (the greatest number) is all the way to the right.

We can now list the temperatures from coldest to warmest:

$$-7, -6, -2, -1, 0, 3, 5$$

b. Sophia is incorrect. It is common for students to compare negative numbers as if they were positive and to assume that the one with the greatest magnitude is the greatest number. However, -23 is to the left of -14 on the number line, and so it is less than -14. Thus

$$-23 < -14$$

and Anchorage was colder.

c. Again, the coldest temperature corresponds to the least number. So the warmest temperature corresponds to the greatest number. Since

$$-55 > -89$$

the average temperature on Mars is warmer than the coldest temperature on Earth.

Misconceptions:
1) Some students are not able to visualize a horizonal number line as a vertical number line such as a thermometer. Once this is visualized, then the students make the comnnections about the larger negative number being the colder temperture.
2) Relating this task back to absolute value. When this is accomplished then the reasoning for -7 being warmer than -9 makes sense. The closer the distance is from zero the warmer the temperature.
3) Defining an inequality as the difference in degrees. The two temperatures are different and not equal to each other. One temperature may be colder than another temperature.

Assessing Questions for the Task:
1) Write two inequalities to support your ordering for the tempertures of Alaska.
2) How can you change Sophie's answer to be true? Explain your reasoning
3) Can you explain the relationship between the magnitude of a number and its temperture?

Advancing Question for the Task:
1) Explain if this inequality statement about temperture is always true, the coldest temperture is the least number, so the greatest number is always the warmest temperture.
2) Explain the similarities and differences of a horizonal and vertical number line.

Illustrative Mathematics

6.NS Jumping Flea

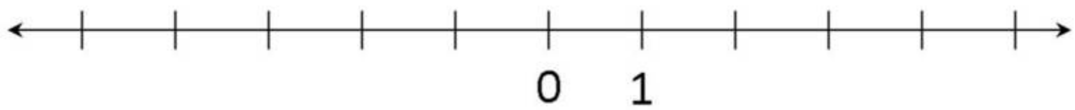
Alignments to Content Standards

- Alignment: 6.NS.C.7

Tags

- *This task is not yet tagged.*

A flea is jumping around on the number line.



a. If he starts at 1 and jumps 3 units to the right, then where is he on the number line? How far away from zero is he?

b. If he starts at 1 and jumps 3 units to the left, then where is he on the number line? How far away from zero is he?

c. If the flea starts at 0 and jumps 5 units away, where might he have landed?

d. If the flea jumps 2 units and lands at zero, where might he have started?

e. The absolute value of a number is the distance it is from zero. The absolute value of the flea's location is 4 and he is to the left of zero. Where is he on the number line?

Commentary

This purpose of this task is to help students understand the absolute value of a number as its distance from 0 on the number line. The context is not realistic, nor is meant to be; it is a thought experiment to help students focus on the relative position of numbers on the number line.

Solutions

Solution: Solutions

It would be a good idea to use a number line to illustrate these solutions,
Have the students genrate their own number lines to support their reasoning.

a. If he starts at 1 and jumps 3 units to the right, then the flea is at 4. He is 4 units away from zero.

b. If he starts at 1 and jumps 3 units to the left, then the flea is at -2. He is $|-2| = 2$ units away from zero.

Misconceptions:
Because the flea landed on 4 when moving to the right of the number line, when the question is asked where would the flea land if it moved 3 jumps to the left from 1, students will have a tendency to say -4. They will not consider O as a number on the number line.

c. If the flea starts at 0 and jumps five units away, then he is either at -5 or 5.

d. If the flea lands on 0 and jumped 2 units, then he started at either -2 or 2.

Misconception:
The flea jumps 2 units and lands on 0, students will assume there is only one answer, 2, when actually -2 would be an answer as well. On the other hand they may choose -2 and forget 2 because they assume the flea cannot jump backwards. This is the first time student will explore the concept of positive and negative numbers and will not consider more than one answer.

e. If the absolute value of the flea's location is 4, then he is either at -4 or 4. Since he is to the left of zero, the flea is at -4.

Assessing Questions:

1) If the flea jumped 4 forward from zero and then started again at 0 and jumped 4 backwards, what would the absolute value be? (4 because it is the distance from zero)

2) If the flea jumped backwards 25 times from 1, where would he land? (-24) How far from zero? (24)

*3)Explain what is absolute value or how this lesson builds understanding of absolute value. This is a good question for the teachers.

Advancing Questions:

1) If the flea started at -1 and jumped 10 to the right and 17 to the left where did he land? What is the distance from zero? Explain your reasoning. (-1 forward 10 equals 9, 9 back jumps to zero then 8 more equal -8. The absolute value is 8 because it is 8 units from zero.

2) Write a probem where the flea jumpes forward and backward on the number line and the absolute value is 11.

3) What is the purpose of teaching absolute value?

Illustrative Mathematics

7.NS Distances Between Houses

Alignments to Content Standards

- Alignment: 7.NS.A.1

Tags

• *This task is not yet tagged.*

Aakash, Bao Ying, Chris and Donna all live on the same street as their school, which runs from east to west.

- Aakash lives $5\frac{1}{2}$ blocks to the west.
- Bao Ying lives $4\frac{1}{4}$ blocks to the east.
- Chris lives $2\frac{3}{4}$ blocks to the west.
- Donna lives $6\frac{1}{2}$ blocks to the east.

a. Draw a picture that represents the positions of their houses along the street.

b. Find how far is each house from every other house?

c. Represent the relative position of the houses on a number line, with the school at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers.

d. How can you see the answers to part (b) on the number line? Using the numbers (some of which are positive and some negative) that label the positions of houses on the number line, represent these distances using sums or differences.

Commentary

The purpose of this task is for students to solve a problem involving distances between objects whose positions are defined relative to a specified location and to see how this kind of situation can be represented with signed numbers.

A full solution requires systematic listing of pairs of houses, a valid list of differences - either subtracting the smaller number from the larger, or taking absolute value of difference in any order - as well as computations which could be done by counting on the number line or subtracting fractions. Note as well the "twist" with the houses not in "alphabetical" order on the number line requires students to make sense of the problem (MP 1)attend to precision (MP 6).

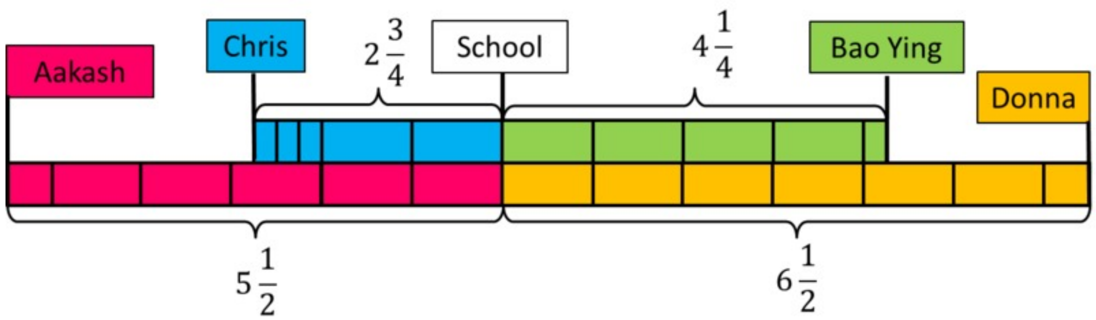
This problem framework could be used with integer values (say to enhance understanding of 6.NS.5) or later for tasks involving the Pythagorean Theorem in the plane (8.G.8).

Solutions

The tape diagram is the best strategy to use to illustrate the distance between houses. Solutions for the distance between all houses should be shown on a picture model, not a number line yet (that is part c.) Question: Can you illustrate this task using a tape diagram?

Solution: 1

a. There are many ways to draw a picture that represents this situation. Here is one:



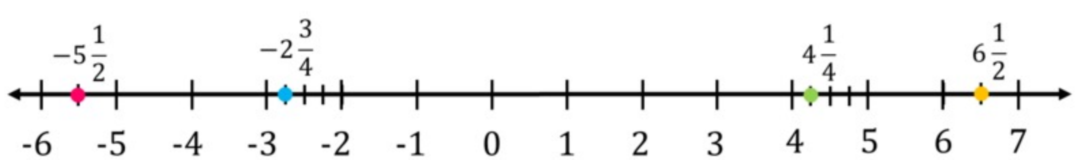
This is just one strategy to display the distance between the houses. However the data is displayed, it needs to be in a form that is easy to read, not just a bunch of problems solved.

b. Here is a table that shows the distances between each of the student's houses.

	Bao Ying	Chris	Donna
Aakash	$9\frac{3}{4}$	$2\frac{3}{4}$	12
Bao Ying		7	$2\frac{1}{4}$
Chris			$7\frac{1}{4}$

Question: How can this information be represented in a table?

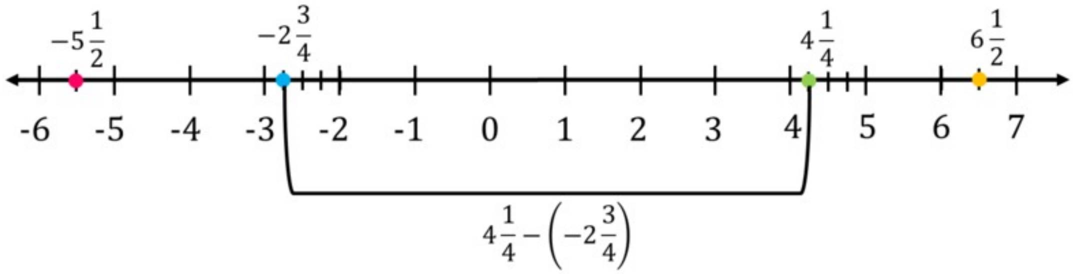
c. The colors show which point corresponds to which person in the first picture:



d. The distance between the houses is represented by the distance between the points that correspond to the houses on the number line. This can be computed by subtracting the numbers that represent the position of the house relative to the school. For example, to find the distance between Bao Ying and Chris, we subtract $-2\frac{3}{4}$ from $4\frac{1}{4}$:

$$4\frac{1}{4} - (-2\frac{3}{4})$$

We can communicate this more clearly by labeling the distance between the points with the difference of the numbers on the number line:



This is the higher level of thinking you want to develop from the model and table, then move to the number line.

Assessing Questions for the Task:
1) How is a number line and a tape diagram similiar? Different?
2) Why is it important to begin with a picture before moving to a number line?
3) How does modeling help in the understanding of the algorithm for adding and subtracting fractions?

Advancing Question for the task:
1) How is the concept of absolute value used to solve this task?

2) If another house was built and it was 2 1/8 blocks east, how would this effect the distances from the other houses? Would your reasoning be true for all new houses built?

Illustrative Mathematics

8.NS Comparing Rational and Irrational Numbers

Alignments to Content Standards

- Alignment: 8.NS.A.2

Tags

• *This task is not yet tagged.*

For each pair of numbers, decide which is larger without using a calculator. Explain your choices.

a. π^2 or 9

b. $\sqrt{50}$ or $\sqrt{51}$

c. $\sqrt{50}$ or 8

d. -2π or -6

Commentary

This task can be used to either build or assess initial understandings related to rational approximations of irrational numbers.

Misconceptions

- Is π rational or irrational? Explain.
- What is the value of π? ($\pi \approx 3.1416$)
- Be sure students square the values correctly.
- Students may have trouble understanding what square root means?

  Ex. the square root of 9 is 3, where three can be squared (multiplied by itself) to get 9
- What does the -2 in front of -2π mean? Explain.

Solutions:

a. $\pi > 3$ so $\pi^2 > 9$. **$\pi \approx 3.1416$ which is greater than 3 so (3.1416)2 = (3.1416)(3.1416) = 9.86960 is greater than (3)2 =(3)(3) = 9.**

b. $\sqrt{50} < \sqrt{51}$ because $50 = (\sqrt{50})^2 < (\sqrt{51})^2 = 51$. **$(50)(50) = (50)^2$**

c. $7^2 = 49$ and $8^2 = 64$. Thus we have that $\sqrt{49} < \sqrt{50} < \sqrt{64}$. So $\sqrt{50} < 8$.

d. $\pi > 3$ so $2\pi > 2 \cdot 3$. If you look at these numbers on the number line, that means that 2π is farther to the right than 6. When you look at their opposites, -2π will be farther to the left than -6, so $-2\pi < -6$.

Question for problem a:

1.What do you know about the value of π? How does this relate to the value of π given in the problem?
2.How does the value of π relate to the 9 in the problem? Explain.
3.Is there a pattern in the values of the problem? Explain.

Questions for problem b:

1.What does it mean to say √50 or √51 ? What is the process for coming up with this value without using a calculator? Explain.
2.What do you know about the values of 50 and 51? How can this help you as you think about the question asked? Explain.
3.Can you write equivalent values for 50 and 51 using without using a calculator? If so, how would that look? Explain.

Questions for problem c:

1.What do you know about these two numbers? Explain.
2.What does each representation mean? Explain.
3.Can these numbers be represented in the same format? Explain.
4.Is there a pattern represented with these two numbers? Explain.
5.What does the pattern say about the two numbers? Explain.

Questions for problem d:

1.What do you know from question a about?
2.Does this have any relationship to the value of -2π in the problem? Explain.
3.Where would the values in the problem be on the number line? How do you know? Explain.

Illustrative Mathematics

8.NS Converting Repeating Decimals to Fractions

Alignments to Content Standards

- Alignment: 8.NS.A.1

Tags

- *This task is not yet tagged.*

Leanne makes the following observation:

> I know that
>
> $$\frac{1}{11} = 0.0909\ldots$$
>
> where the pattern 09 repeats forever. I also know that
>
> $$\frac{1}{9} = 0.1111\ldots$$
>
> where the pattern 11 repeats forever. I wonder if this is a coincidence?

a. What is the decimal expansion of $\frac{1}{99}$? Use this to explain the patterns Leanne observes for the decimals of $\frac{1}{9}$ and $\frac{1}{11}$.

b. What is the decimal expansion of $\frac{1}{999}$? Use this to help you calculate the decimal expansions of $\frac{1}{27}$ and $\frac{1}{37}$. How does this relate to Leanne's observations?

Commentary

The purpose of this task is to study some concrete examples of repeating decimals and find a way to convert them to fractions. The key observation is to use the fractions $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, …. These fractions all have particularly simple decimal expansions which can be used to write repeating decimals as fractions.

This task leads naturally to a simple method for writing repeating decimals as fractions which the teacher may wish to explore. This technique is very closely related to the denominators 99, 999, …. Suppose we have a repeating decimal such as

$$0.137137\ldots$$

where the three digits 137 go on forever. The ideas presented in this task suggest that this decimal number is equal to $\frac{137}{999}$. A second, important way to see this uses an algebraic argument. If we set $x = 0.137137\ldots$ then $1000x = 137.137137$… . We can then perform subtraction:

$$\begin{array}{r} 137\ .137137\ldots \\ -\ 0\ .137137\ldots \end{array}$$

The result of the subtraction is 137 since the numbers after the decimal all cancel out. This show that

$$1000x - x = 137.$$

Solving for x gives $x = \frac{137}{999}$.

The algebraic approach of the previous paragraph and the arithmetic approach in the solution are both important. The reason why the arithmetic approach is adapted in the solution here is that Leanne already knows what the fraction representation of these decimals are and she is interested in explaining the pattern that she observes. The explanation for the pattern turns out to be a key for writing *any* repeating decimal as a fraction.

Solutions

Solution: 1

a. The decimal expansion of $\frac{1}{99}$ is 0.0101… where the pattern 01 repeats forever. From the point of view of the long division algorithm, 99 does not go into 1 or into 10 but it goes into 100 with a remainder of 1. We add two more zeroes and then 99 goes in once again with a remainder of 1. This pattern goes on forever. We have

$$\begin{aligned} \frac{1}{11} &= \frac{9}{99} \\ &= 9 \times \frac{1}{99} \\ &= 9 \times 0.0101\ldots \\ &= 0.0909\ldots \end{aligned}$$

In order to justify the last step, we can look at the division algorithm. Just as for $\frac{1}{99}$, when we calculate $\frac{9}{99}$ we find that 99 does not go into 9 or into 90 but it does go into 900 nine times with a remainder of 9. We will bring down two more zeroes and then 99 will again go in nine times with a remainder of 9. This gives us the decimal expansion 0.0909… above.

The same methods work for the decimal of $\frac{1}{9}$:

$$\begin{aligned} \frac{1}{9} &= \frac{11}{99} \\ &= 11 \times \frac{1}{99} \\ &= 11 \times 0.0101\ldots \\ &= 0.1111\ldots \end{aligned}$$

The last step in these calculations can be explained as above. When we perform long division to find $\frac{1}{9}$, 9 does not go into 1 but it goes into 10 once with a remainder of 1. We add a zero and 9 again goes in once with a remainder of 1. This pattern goes on forever.

b. The decimal expansion of $\frac{1}{999}$ works just like $\frac{1}{99}$ except that we need to add three zeroes after the decimal before 999 goes in once with a remainder of 1. We then add three more zeroes and 999 goes in once again with a remainder of 1. This pattern continues forever and so

$$\frac{1}{999} = 0.001001\ldots$$

Just as $9 \times 11 = 99$, the numbers 27 and 37 have been chosen because $27 \times 37 = 999$. Working as above we have

$$\begin{aligned} \frac{1}{27} &= \frac{37}{999} \\ &= 37 \times \frac{1}{999} \\ &= 37 \times 0.001001\ldots \\ &= 0.037037\ldots \end{aligned}$$

To justify the last step in this calculation note that 999 does not go into 37 or into 370 but it does go into 3700 three times with a remainder of 703. Then 999 goes into 7030 seven times with a remainder of 37. This is where we started and so the pattern 037 repeats forever in this decimal.

We can argue similarly for $\frac{1}{37}$:

$$
\begin{aligned}
\frac{1}{37} &= \frac{27}{999} \\
&= 27 \times \frac{1}{999} \\
&= 27 \times 0.001001\ldots \\
&= 0.027027\ldots
\end{aligned}
$$

The last step in the calculation is justified in the same way as for $\frac{1}{37}$.
Leanne noticed that $9 \times 11 = 99$ and the decimals for the unit fractions are

$$
\begin{aligned}
\frac{1}{9} &= 0.1111\ldots \\
\frac{1}{11} &= 0.0909\ldots
\end{aligned}
$$

So the repeating part of $\frac{1}{9}$ is 11 and the repeating part of $\frac{1}{11}$ is 09. For $\frac{1}{27}$ and $\frac{1}{37}$ we have a similar phenomenon:

$$
\begin{aligned}
\frac{1}{27} &= 0.027027\ldots \\
\frac{1}{37} &= 0.037037\ldots
\end{aligned}
$$

Again the repeating part of $\frac{1}{27}$ is 037 and the repeating part of $\frac{1}{37}$ is 027. Experimentation with other factorizations of 999 (or more 9's) will show the same behavior. For example, $9999 = 99 \times 101$ and

$$
\begin{aligned}
\frac{1}{99} &= 0.01010101\ldots \\
\frac{1}{101} &= 0.00990099\ldots
\end{aligned}
$$

Illustrative Mathematics

8.NS Irrational Numbers on the Number Line

Alignments to Content Standards

- Alignment: 8.NS.A.2

Tags

- *This task is not yet tagged.*

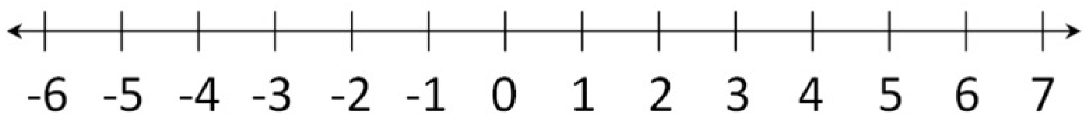
Without using your calculator, label approximate locations for the following numbers on the number line.

a. π

b. $-(\frac{1}{2} \times \pi)$

c. $2\sqrt{2}$

d. $\sqrt{17}$



2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). [8-NS2]

When students plot irrational numbers on the number line, it helps reinforce the idea that they fit into a number system that includes the more familiar integer and rational numbers. This is a good time for teachers to start using the term "real number line" to emphasize the fact that the number system represented by the number line is the real numbers. When students begin to study complex numbers in high school, they will encounter numbers that are not on the real number line (and are, in fact, on a "number plane"). This task could be used for assessment, or if elaborated a bit, could be used in an instructional setting.

Solutions

Solution: Solution

a. π is slightly greater than 3. **$(3)^2 = 9$ or $(3.14)^2 \approx \pi$**

b. $-(\frac{1}{2} \times \pi)$ is slightly less than -1.5.

c. $(2\sqrt{2})^2 = 4 \cdot 2 = 8$ and $3^2 = 9$, so $2\sqrt{2}$ is slightly less than 3.

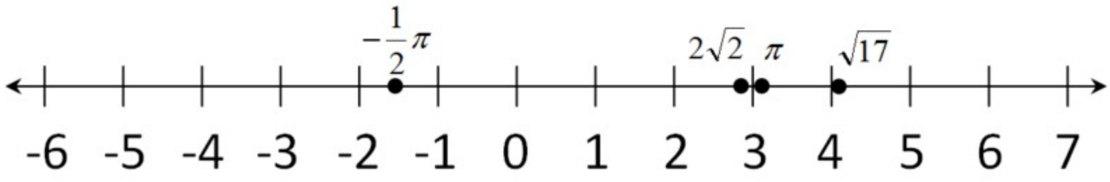
d. $\sqrt{16} = 4$, so $\sqrt{17}$ is slightly greater than 4.

Questions for problem b:
1.How did you place π in question a? Explain.
2.What do you think the value of question b is? How did you come up with this answer? Explain.
3.What is different about this question and question a? How will this change where you put this value on the number line?

Question problem c:
1.What two numbers do you think this value will lie between? Explain?
2.If you square these two numbers, how will they look? Explain. How can this help in the estimation of 2√2 ? Explain.
3.What whole number is 2√2 closes to? Explain.

Questions for problem d:
1.What two whole numbers does this fall between? Explain.
2.Which of these numbers is √17 closes to? How do you know? Explain.

Misconceptions:
- What does π mean?
- How does multiplying a 1/2 change the value of π? - How does the negative change where the number is on the number line?
- What a perfect square is?
-The relationship between perfect squares and square roots?



Illustrative Mathematics

N-RN Extending the Definitions of Exponents, Variation 2

Alignments to Content Standards

- Alignment: F-LE.A
- Alignment: N-RN.A.1

What assessing and advancing questions might you ask?

Does your equation work with all the values in your table? Tell me about your graph.

Tags

- *This task is not yet tagged.*

A biology student is studying bacterial growth. She was surprised to find that the population of the bacteria doubled every hour.

a. Complete the following table and plot the data.

Hours into study	0	1	2	3	4
Population (thousands)	4				

b. Write an equation for P, the population of the bacteria, as a function of time, t, and verify that it produces correct populations for t = 1, 2, 3, and 4.

c. The student conducting the study wants to create a table with more entries; specifically, she wants to fill in the population at each half hour. However, she forgot to make these measurements so she wants to estimate the values.

Instead she notes that the population increases by the same factor each hour, and reasons that this property should hold over each half-hour interval as well. Fill in the part of the table below that you've already computed, and decide what constant factor she should multiply the population by each half hour in order to produce consistent results. Use this multiplier to complete the table:

Hours into study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
Population (thousands)	4						

d. What if the student wanted to estimate the population every 20 minutes instead of every 30 minutes. What multiplier would be necessary to be consistent with the population doubling every hour? Use this multiplier to complete the following table:

Hours into study	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
Population (thousands)	4						

e. Use the population context to explain why it makes sense that we define $2^{\frac{1}{2}}$ to be $\sqrt{2}$ and $2^{\frac{1}{3}}$ as $\sqrt[3]{2}$.

f. Another student working on the bacteria population problem makes the following claim:

If the population doubles in 1 hour, then half that growth occurs in the first half-hour and the other half occurs in the second half-hour. So for example, we can find the population at $t = \frac{1}{2}$ by finding the average of the populations at $t = 0$ and $t = 1$.

Comment on this idea. How does it compare to the multipliers generated in the previous problems? For what kind of function would this reasoning work?

Commentary

The goal of this task is to develop an understanding of why rational exponents are defined as they are (N-RN.1), however it also raises important issues about distinguishing between linear and exponential behavior (F-LE.1c) and it requires students to create an equation to model a context (A-CED.2) Have each group post their task on chart paper. When all the tasks are posted, participants will discuss the progression of number through the grades (6 – HS)

Solutions

Be sure to include graph paper in your materials.

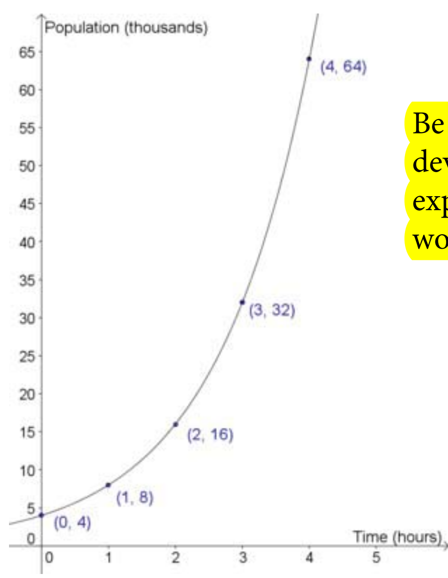
Solution: Solution

Push participants to work this task through letter e so that they will have to write an explanation for the meaning of the number in the exponent.

a.

Hours into study	0	1	2	3	4
Population (thousands)	4	8	16	32	64

Students would be expected to find these values by repeatedly multiplying by 2. The plot below consists of the exponential function $P(t) = 4 \cdot 2^t$ which students will derive in the next part. The plot of the data alone would consist of the 5 plotted blue points.



Be sure to discuss the fact that this task develops extending the definitions of exponents through application or real-world problems.

b. The equation is $P(t) = 4 \cdot 2^t$, since as we tallied above via repeated multiplication, we have $4 \cdot 2^1 = 8$, $4 \cdot 2^2 = 16$, $4 \cdot 2^3 = 32$, $4 \cdot 2^4 = 64$, etc.

c.

Hours into study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
Population (thousands)	4	5.657	8	11.314	16	22.627	32

Let x be the multiplier for the half-hour time interval. Then since going forward a full hour in time has the effect of multiplying the population by x^2, we must have $x^2 = 2$, and so the student needs to multiply by $\sqrt{2}$ every half hour.

d.

Hours into study	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
Population (thousands)	4	5.010	6.350	8	10.079	12.699	16

Similarly, since waiting three 20-minute intervals should double the population, the new multiplier has to satisfy $x \cdot x \cdot x = 2$, which gives $x^3 = 2$. So you would need to multiply by $\sqrt[3]{2}$ every 20 minutes to have the effect of doubling every hour.

e. We already know that the equation for population is $P = 4(2)^t$ when t is a natural number. Given this, it's reasonable to use the expression $P(\frac{1}{2}) = 4(2)^{\frac{1}{2}}$ to define $2^{\frac{1}{2}}$. However, we reasoned above that $P(\frac{1}{2}) = 4 \cdot \sqrt{2}$, and equating the two gives $2^{\frac{1}{2}} = \sqrt{2}$. Similarly, equating the expression $P(\frac{1}{3}) = 2^{1/3}$ with the calculation $P(\frac{1}{3}) = \sqrt[3]{2}$ gives the reasonable definition $2^{1/3} = \sqrt[3]{2}$. Make sure an explanation is written. This part of the problem is key to the understanding of developing an understanding of number in the exponent.

f. The reasoning mistakenly assumes linear growth within each hour, i.e., that the amount of population growth is the same each half hour. We know instead that the percentage growth is constant, not the raw change in population. If we were to apply the faulty reasoning to the first hour, we would get the following values:

Hours into study	0	$\frac{1}{2}$	1
Population (thousands)	4	6	8

However, this does not have constant percentage growth: from $t = 0$ to $t = \frac{1}{2}$ this population grew by 50% $(\text{ratio} = 1.5)$, but then from $t = \frac{1}{2}$ to $t = 1$ the ratio is only 1.33. If you graphed this data, instead of seeing a smoothly increasing curve, you would see a series of connected line segments of increasing slopes.