Operations and Algebraic Thinking Tasks and Answer Key
Attached you will find a set of 6 tasks for Operations and Algebraic Thinking. These tasks are interchangeable with the practice items. If your students, or a group of students, are ready for problems that are a bit more rigorous, feel free to use the tasks. The tasks are best used by partners or small groups. Since these tasks are more in-depth than the practice items, you would not want to use them every day. It is not unusual to spend the entire math class on a high-level task. You could use them once or twice a week, or if they address a standard you are working on that day, you could use them as the foundation of your lesson.

The purpose of using tasks is to help you see how students solve problems, and understand their thought process while they work. Students working with others and engaging in productive discourse, explaining their thinking to another student, and developing a solution, is the most effective way to get to student understanding. These tasks require students to do just that: think about an efficient strategy to solve the problem, show their work and justify their reasoning. This is the ultimate goal for what we want students to be able to do. Being able to gather evidence of student learning and misconceptions in the moment, will give you the flexibility to change your instruction to meet their needs. As the instructional decision-maker, you are able to adjust your methods for whole class or small groups to address student misconceptions and move them toward proficiency.

The goal is to have tasks that can be interchanged with the practice items when needed. There are tasks that represent the 5 domains in 4th grade. We would like for you to use these tasks along with the practice items for a 10 week period between the time you receive them and the end of January.

At the end of each task packet, you will find an answer key for your use. Some tasks include possible responses that students might have on the constructed response items.

When implementing the tasks with your students, please take the time to have the students read through the tasks before starting to see if they have any questions about vocabulary or what the task is asking them to do. Taking the time to do these things now, will help assure that the students are familiar with mathematical vocabulary and different question types before the actual test.

Since the purpose of the tasks and practice items is to get at student understanding, it is not enough just to give them as bell ringers or engagement items. A key part of the process in advancing student thinking is to debrief the tasks and provide specific feedback on the student’s thinking and performance. The key to getting at student understanding and thinking is to always have them explain how they solved the problem. This can be done during the sharing out process by asking effective questions. It is difficult to make student thinking and understanding visible by just letting students solve the problems and determining whether their response is correct or incorrect. Asking questions similar to the ones below can help students verbalize the reasoning for their solutions:

- To solve the problem, what concept do you have to be aware of?
- Why do you think your solution is correct?
- What strategy did you use to solve the problem? Why did you use that particular strategy?
• Is there another strategy that you could use to solve the problem?

The above questions can be used with short response and constructed response also. Other questions to consider when prompting students to verbalize or justify their thinking are:

**Monitoring as students work:**

• What is the problem asking you to find?
• How would/did you start the problem?
• What else do you need to do?

**During debriefing:**

• What did the problem ask you to do?
• What information do you see in the problem?
• What did you do first to solve this problem?
• Who else started this same way?
• What did you do next?
• Who started a different way?
• What are some strategies that you heard today that you would like to try when solving a similar problem in the future?

Another option is to let the groups draw out their solution(s) to the task on chart paper, or use a document camera to display and explain their thinking to the class. They can then share out with the whole class. With this option the students are able to present their thinking, justify their reasoning, and answer questions from the other students.

**Answer Key:**

The information above is intended to help teachers get at student understanding of the mathematical idea(s) in each problem. Also provided is an Answer Key for each set of tasks. The Answer Key provides more information on the expected student response for each task, as well as the standard being addressed. While it is important for students to get the answer right, it is equally important for them to understand how their thinking leads or does not lead to a correct solution. Incorrect solutions set the stage for teachable moments!!!!
Task 1 – Standard #2

Comparing Money Raised

Part A:

Helen raised $12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year? Use a model or an equation to justify your answer.

Part B:

Sandra raised $15 for the PTA and Nita raised $45. How many times as much money did Nita raise as compared to Sandra? Use a model or an equation to justify your answer.

Part C:

Luis raised $45 for the animal shelter, which was 3 times as much money as Anthony raised. How much money did Anthony raise? Use a model or an equation to justify your answer.

Task from Illustrative Mathematics
Task 2 – Standard #3

How Many Teams?

In eastern North Carolina there are 3,277 fourth graders signed up for basketball. In western North Carolina there are 2,981 fourth graders signed up for basketball. In the Piedmont region there are 1,512 players signed up. Every player will get placed on a team in their region of the state.

Part 1:
The league wants to place 9 players on each team? Leftover players will be added to teams, so some teams will have ten players. How many teams will have 9 players in each region of the state? How many teams will have 10 players in each region of the statewide? Statewide, how many teams have 9 players and how many teams have 10 players? Show your work and explain your reasoning.

Part 2:
In order to maximize playing time, the league decides to only place 7 players on each team. If there are extra players, some teams will have 8 players. How many teams will have 7 players in each region of the state? How many teams will have 8 players in each region of the state? Statewide, how many teams have 7 players and how many teams have 8 players? Show your work and explain your reasoning.
Task 3 – Standard #4

A Ride On A Bus

Part 1:

Eighty fourth grade students at Andrews Elementary School are going on a field trip. Their teachers need to put between 3 and 25 students in each group to visit the shark tank. How many different ways can the teachers group their students so that each group has the same number of students?

Part 2:

If four groups of eight students ride bus 1, how many students will ride bus 2?

How many different ways can the teacher group the students on bus 2 so that each group has the same number of students? Explain your reasoning using pictures, numbers or words.
Task 4 – Standard #4

Arranging Chairs

Part 1:
There are 24 chairs in the art room. What are the different ways that the chairs can be arranged into equal groups if you want at least 2 groups and want at least 2 chairs in each group? How do you know that you have found every arrangement? Write division equations to show your answers.

Part 2:
There are 48 chairs in the multi-purpose room. What are the different ways that the chairs can be arranged into equal groups if you want at least 2 groups and want at least 2 chairs in each group? How do you know that you have found every arrangement? Write division equations to show your answers.

Part 3:
What relationship do you notice about the size of the groups if the chairs were arranged in 4 groups in both Part 1 and Part 2? What about if the chairs were arranged in 8 groups? Explain why you think this relationship exists.
Table Dilemma

Square tables at Giovanni’s Pizza seat 4 people each. For bigger groups, square tables can be joined. Tables can be pushed together so that they share a side.

Part 1:

One square table seats 4 people.
Two square tables seat 8 people.
How many people can sit at 3 tables? 4 tables? 5 tables?

Make a chart to show how many people can be seated at five tables that are not pushed together. Find a rule that helps you predict the number of people that can be seated at $n$ tables.

Part 2:

Two tables pushed together seat 6 people.
How many people can sit at three tables pushed together? 4 tables pushed together?
5 tables pushed together?

Make a chart to show how many people can be seated at five tables that are pushed together. Find a rule that helps you predict the number of people that can be seated at $n$ tables.

Part 3:

Compare the patterns that you see on your charts. What pattern do you notice for each chart?
Task 6 – Standard #5

**Lawn Mowing Business**

Ted and Nancy both mow lawns during the summer to earn money.
Ted charges $10 per lawn and $2 per hour.
Nancy charges $4 per lawn and $4 per hour.

**Part 1:**
Complete the table to show how much Ted and Nancy would each earn based on the amount of time that it took to mow a lawn.

<table>
<thead>
<tr>
<th>Time</th>
<th>Ted</th>
<th>Nancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>½ hour</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 hour</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 and ½ hours</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 hours</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 and ½ hours</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 hours</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 and ½ hours</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 hours</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Part 2:**
Explain how you found the amount for both Ted and Nancy in the table.

**Part 3:**
Last week, Ted and Nancy each mowed 3 lawns that took 1 and ½ hours each, and 7 lawns that took 3 hours each. How much money did they each earn? Write an equation to show your work.
ANSWER KEY

Task 1: Comparing Money Raised

Purpose:
The purpose of this task is for students to solve three comparisons problems that are related by their context but are structurally different. Multiplicative comparison is purposefully excluded from third grade (see 3.OA.3 and 3.MD.2), making this task appropriate for fourth but not third grade.

In these multiplicative comparison problems, one factor and the product are amounts of money and the other factor represents the number of times bigger one amount is than the other. Sometimes this second factor is called a “scale factor.” In part (a), the larger amount (which is the product) is unknown, while in part (b) the scale factor is unknown and in part (c) the smaller amount of money is unknown. Students will study multiplicative comparison problems involving scale factors that are fractions in 5th grade; see 5.NF.B.5. Note that in fourth grade, scale factors must always be bigger than 1, so students often think that “multiplying makes bigger”; however in 5th grade they will see that when the scale factor is less than 1, the product will actually be smaller than the initial quantity.

Note that the numbers in parts (b) and (c) are related by the fact family. This allows for a classroom discussion about the different interpretations of the factors in a multiplicative comparison context.

Solutions:
A. She raised six times as much money (as shown in the diagram), so she raised \(6 \times 12 = 72\).

\[
\begin{align*}
\text{Money she raised last year:} & \quad \underline{12} \\
\text{Money she raised this year:} & \quad \underline{12} \quad \underline{12} \quad \underline{12} \quad \underline{12} \quad \underline{12} \\
& \quad ?
\end{align*}
\]

Helen raised $72 this year.

B. This is a “Number of Groups Unknown” problem.

\[? \times 15 = 45\] is equivalent to \(45 \div 15 = ?\)

\[
\begin{align*}
\text{Money Sandra raised:} & \quad \underline{15} \\
\text{Money Nita raised:} & \quad \underline{\ldots \ ? \ \ldots} \\
& \quad 45
\end{align*}
\]

Nita raised 3 times as much as Sandra.

C. This is a “Group Size Unknown” problem.

\[3 \times ? = 45\] is equivalent to \(45 \div 3 = ?\)

\[
\begin{align*}
\text{Money Anthony raised:} & \quad ? \\
\text{Money Luis raised:} & \quad \underline{\ldots \ ? \ \ldots} \\
& \quad 45
\end{align*}
\]

Anthony raised $15.
**Task 2: How Many Teams**

Depending on students’ ability to work with four-digit dividends and when this task is presented, the number of players in each region can easily be modified to three- or even two-digit numbers. Alternative dividends are: 327, 298, and 151. These numbers are chosen so that students interpret remainders, as in the original problem. Using alternate numbers for each dividend may be appropriate for the whole class or for a small group of students. Using modified numbers still allows all students to work with and practice grade-level problems.

It is important to note that this task does not address the second or third parts of the standard, representing problems using equations with a letter standing for the unknown and assessing the reasonableness of the answers using mental computation and estimation strategies including rounding. The task can be modified to more fully address the standard by asking students to round or use compatible numbers to estimate the quotient before completing the computation.

**Solutions:**

Part 1:

East: 363 teams have 9 players. 1 team has 10 players.
West: 329 teams have 9 players. 2 teams have 10 players.
Piedmont: 168 teams have 9 players.
No teams have 10 players.

Part 2:

East: 467 teams have 7 players. 1 team has 8 players.
West: 419 teams have 7 players. 6 teams have 8 players.
Piedmont: 216 teams have 7 players.
No teams have 8 players.
Statewide: 1,102 teams have 7 players

**Task 3: A Ride on the Bus**

- Task 1: Example: 4 students in a group, 20 groups
  5 x 16 and 8 x 10

- Task 2: Bus 2 has a total of 48 students,
  Possible groupings: 1 x 48, 2 x 24, 3 x 16, 4 x 12, 6 x 8
Task 4: Arranging Chairs

Part 1:
2 groups of 12
3 groups of 8
4 groups of 6
6 groups of 4
8 groups of 3
12 groups of 2

Part 2:
2 groups of 24
3 groups of 16
4 groups of 12
6 groups of 8
8 groups of 6
12 groups of 4
16 groups of 3
24 groups of 2.

Part 3:
24 chairs can be put into 4 groups of 6.
48 chairs can be put into 4 groups of 12.
24 chairs can be put into 8 groups of 3.
48 chairs can be put into 8 groups of 6.
The explanation should reference that 48 is double 24, which means that when one factor remains constant, the other factor is doubled.

Task 5: Table Dilemma

Part 1: Students will create a chart to show tables: people (perimeter)

1 table: 4 people
2 tables: 8 people
3 tables: 12 people
4 tables: 16 people
5 tables: 20 people
They will identify the rule as n x 4.
Part 2: Students will create a chart to show tables: people (perimeter)

1 table: 4 people
2 tables: 6 people
3 tables: 8 people
4 tables: 10 people
5 tables: 12 people

They will identify the rule as \( n \times 2 + 2 \).

- Observations may include:
  Numbers in chart 1 increase by 4 vertically while numbers in chart 2 increase by 2 vertically. All numbers (of seats) are even. **AND** Students write a clear and accurate explanation of both patterns in Part 3.

Task 6: Lawn Mowing Business

Part 1:

<table>
<thead>
<tr>
<th></th>
<th>Ted</th>
<th>Nancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>½ hour</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>1 hour</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>1 and ½ hours</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>2 hours</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>2 and ½ hours</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>3 hours</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>3 and ½ hours</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>4 hours</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

Part 2:

The explanation is clear and accurate.

Part 3:

Ted: \( 3 \times 13 + 7 \times 16 = 151 \)
Nancy: \( 3 \times 10 + 7 \times 16 = 142 \)