Grade 6 Standards

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio, and use ratio language to describe a ratio relationship between two quantities. [6-RP1]
   Examples: "The ratio of wings to beaks in the bird house at the zoo was 2:1 because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

2. Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. [6-RP2]
   Examples: "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. [6-RP3]
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. [6-RP3a]

Essential Understandings (Mathematical Goals)

- A ratio can compare a part of a quantity to the whole (part-whole), or to another part (part-part). A third type of ratio compares two different things, e.g., miles/hour.
- Reasoning with ratios involves attending to and coordinating two quantities. (*NCTM EU #1*)
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit (*NCTM EU #2*)

Grade 7 Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. [7-RP1]

   Example: If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.

2. Recognize and represent proportional relationships between quantities. [7-RP2]
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. [7-RP2a]
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. [7-RP2b]
   c. Represent proportional relationships by equations. [7-RP2c]

      Example: If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. [7-RP2d]

3. Use proportional relationships to solve multistep ratio and percent problems. [7-RP3]

   Examples: Sample problems may involve simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

<u>Essential Understandings (Mathematical Goals)</u>

Reasoning with ratios involves attending to and coordinating two quantities.
Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
Linear functions have constant rates of change.

Grade 8 Standards

Understand the connections among proportional relationships, lines, and linear equations.

8. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. [8-EE6]

Define, evaluate, and compare functions.

11. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.) [8-F1]

<u>Essential Understandings (Mathematical Goals)</u>

- Functions provide a tool for describing how variables change together. Using a function in this way is called modeling, and the function is called a model.
- Functions can be represented in multiple ways—in algebraic symbols, situations, graphs, verbal descriptions, tables, and so on—and these representations, and the links among them, are useful in analyzing patterns of change.
- Some representations of a function may be more useful than others, depending on how they are used.
- Linear functions have constant rates of change.

Algebra I Standards

Create equations that describe numbers or relationships. ***(Linear, quadratic, and exponential (integer inputs only); for Standard 14, linear only.)***
12. Create equations ~~and inequalities~~ in one variable, and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

14. Represent constraints by equations ~~or inequalities, and by systems of equations and/or inequalities~~ and interpret solutions as viable or non-viable options in a modeling context. [A-CED3]
Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

Solve equations and inequalities in one variable. ***(Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions.)***
17. Solve linear equations ~~and inequalities~~ in one variable, ~~including equations with coefficients represented by letters~~. [A-REI3]

Interpret functions that arise in applications in terms of the context. (*Linear, exponential, and quadratic.*)
29. Relate the domain of a function ~~to its graph and, where applicable~~, to the quantitative relationship it describes.* [F-IF5]
Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Understand the concept of a function and use function notation. ***(Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences.)***
25. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. ~~The graph of f is the graph of the equation $y = f(x)$~~. [F-IF1]

Build a function that models a relationship between two quantities. ***(For standards 34 and 35, linear, exponential, and quadratic.)***
34. Write a function that describes a relationship between two quantities.* [F-BF1]
a. Determine an explicit expression, a recursive process, or steps for calculation from a context. [F-BF1a]

Essential Understandings (Mathematical Goals)

- Equations can be used to model real-world scenarios. The variables used in the equation must be defined.
- A function assigns to each element of the domain exactly one element of the range.
- Solutions to an equation must be considered viable based on the domain of the function and context of the scenario.

Geometry Standards

Use coordinates to prove simple geometric theorems algebraically. ***(Include distance formula; relate to Pythagorean Theorem.)***

32. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. [G-GPE6]

33. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* [G-GPE7]

<u>Essential Understandings (Mathematical Goals)</u>

- The perimeter of figures whose side lengths are in an n:m ratio will also be in an n:m ratio because addition preserves this ratio.
- Behind every proof is a proof idea.
- Empirical verification is an important part of the process of proving, but it can never, by itself, constitute a proof.
- Geometry uses a wide variety of kinds of proofs.

Algebra II Standards

Write expressions in equivalent forms to solve problems.

14. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]

Example: Calculate mortgage payments.

<u>Essential Understandings (Mathematical Goals)</u>

- The concept of function is intentionally broad and flexible, allowing it to apply to a wide range of situations. The notion of function encompasses many types of mathematical entities in addition to “classical” functions that describe quantities that vary continuously. For example, matrices and arithmetic and geometric sequences can be viewed as functions.
- Functions can be represented in multiple ways, including algebraic (symbolic), graphical, verbal, and tabular representations. Links among these different representations are important to studying relationships and change.
- The convergence or divergence of the geometric series depends on the value of the common ratio.