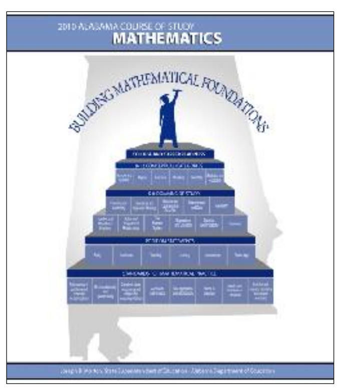
Alabama
College- and
Career-Ready
Standards & Support
Navigating Success for All..

6 - 12

Mathematics Participant Packet



September 2013

The 2012 Quarterly Regional Meetings with local CCRS Implementation Teams

provided an opportunity for LEA teams to experience and be able to transfer to other teachers how to:

Quarterly Meeting # 1

- Build their understanding of the Standards for Mathematical Practice.
- Enhance their skills in identifying the extent to which students exhibit the Standards for Mathematical Practice.
- Generate ideas for how teachers can integrate the Standards for Mathematical Practice with instruction to support student proficiency.
- Explore the Alabama Insight Tool to deepen your understanding of new College and Career-Ready Standards of Mathematics

Quarterly Meeting #2

- Review standards for mathematical practice and reflect on implications for instruction.
- Reflect on criteria used to evaluate mathematics lessons and tasks.
- Determine important criteria to assess mathematics lessons that exemplify College and Career Ready

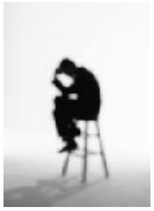
Quarterly Meeting #3

- Discuss and examine the thought process of developing a lesson/unit that is standards-based.
- Select rich tasks that match student friendly outcomes and provide evidence of student learning.
- Develop student friendly outcomes that reflect the rigor and depth of content.
- Model teacher collaboration on identifying resources for implementing College- and Career-Ready Standards for mathematics.

Quarterly Meeting #4

- Identify and evaluate strategies that provide effective instructional support (part II of TTLP – explore) – for the targeted standards and use of precise and accurate mathematics
- Discuss and reach a consensus on what is observable evidence of the degree to which students can independently demonstrate the targeted CCRS standards (hear, see, model, write) (Practice Standards)

Reflection



Year One Reflection

1. What have you changed about your practice in response to implementing the College-and Career-Ready Math Standards?

2. What are two priorities related to implementation of the CCRS Math you have identified for 2013-2014?

3. How has incorporating the College-and-Career-Ready Math Standards into your classroom culture caused your students to learn and behave differently?

Student Discourse

Is	Is Not
Examples	Examples

Unlocking Engagement Through Mathematical Discourse

Best Practices for Student Engagement
January 3, 2013 | Volume **8** | Issue **7**

http://www.ascd.org/ascd-express/vol8/807-miller.aspx

Kirsten Miller

Engaging students in math can be a challenge for teachers. For many students, math is a subject to be endured, not embraced. But the Common Core math standards, which provide a narrower and deeper focus on math than most existing state standards, offer a prime (no pun intended) opportunity to engage students in content through mathematical discourse.

The Common Core State Standards for Mathematical Practice (see sidebar) encourage students to "engage in the actual use of mathematics, not just in the acquisition of knowledge about the discipline" (Schwols & Dempsey, 2012a, p. 7). Research tells us that teaching that focuses on interactive participation can improve problem solving and conceptual mastery, with no ill effects on computational mastery (Bruce, 2007).
Through mathematical discourse, teachers can foster student engagement and participation while focusing on the deep conceptual understanding called for in the Common Core math standards.

Common Core Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Defining Discourse

What is mathematical discourse? At its most basic, mathematical discourse occurs when teachers ask questions and students respond. But there's more to it than that if we're going to get to the level of discourse that encourages students to "think like mathematicians." Schwols and Dempsey (2012b) identify the following components of high-quality mathematical discourse:

- **Questioning.** Questioning comes easily to most teachers. In fact, research suggests that up to 80 percent of teachers' interactions with students include questioning (Fillippone, 1998). During math discourse, questioning should challenge students to be inquisitive and help them extend their existing mathematics knowledge—for example, "Why does this work?" "Is there a more efficient way of doing that?" and "Does this work in every case?" (Schwols & Dempsey, 2012b).

  These types of open-ended questions help foster students' problem solving and increase conceptual understanding (Martino & Maher, 1999)—and increase students' engagement in the subject matter.
- **Facilitation of conversation.** Though student interaction is an important part of mathematical discourse, teachers aren't off the hook. Teachers need to facilitate student discussions by providing a safe and appropriate environment for learning, establishing norms, supporting students throughout their conversations, and staying focused on the students' conceptual understanding, rather than simply focusing on arriving at the right answer (Stein, 2007).

  When students feel safe, respected, and supported, they're more likely to engage in the subject matter—and with their peers.

- **Appreciation for accuracy and reliance on reasoning and proof.** Accuracy is clearly a crucial component of mathematics—and mathematical discourse—but accuracy alone won't let us know whether students have genuinely reached the deep level of conceptual understanding called for in the Common Core math standards.

  Though we often ask students to explain their reasoning when their answers *aren't* accurate, it's equally important for students to explain their reasoning when their answers are. Understanding students' reasoning (right or wrong) provides teachers with valuable information for planning instruction.
- **Collaborative exchange of ideas.** When students come together to discuss math concepts, compare ideas, justify methods, and articulate their thinking, they become more motivated to learn mathematics (Kilpatrick, Swafford, & Findell, 2001).

Pulling in the Practices

When we map these recommendations for high-quality discourse against the mathematical practice standards, we can see immediate connections: for example, engaging in a collaborative exchange of ideas about a mathematics concept provides students with the ideal opportunity to construct viable arguments and critique the reasoning of others (mathematical practice standard #3). Are you fostering an appreciation for accuracy in your conversations with students? Then you're also addressing mathematical practice standard #6, "attend to precision."

The benefits of mathematical discourse go beyond student engagement. But by engaging students in mathematical discourse, we *engage* students—period.

References

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STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices are based on important "processes and proficiencies" that have longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report, *Adding It Up*: *Helping Children Learn Mathematics*. These proficiencies include adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight Standards for Mathematical Practice are listed below along with a description of behaviors and performances of mathematically proficient students.

Mathematically proficient students:

1. Make sense of problems and persevere in solving them.
These students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. These students consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to obtain the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solve complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships. One is the ability to *decontextualize*, to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents. The second is the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

These students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. These students justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments; distinguish correct logic or reasoning from that which is flawed; and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until the middle or upper grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

These students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, students might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, students might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas and can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a Web site, and use these to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
These students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. Mathematically proficient students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. These students also can pause and reflect for an overview and shift perspective. They can observe the complexities of mathematics, such as some algebraic expressions as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
They notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As students work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details and continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The eight Standards for Mathematical Practice described on the previous pages indicate ways in which developing student practitioners of the discipline of mathematics increasingly must engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. It is important that curriculum, assessment, and professional development designers be aware of the need to connect the mathematical practices to the mathematical content standards.

The *Common Core State Standards for Mathematics*, also referred to as the Standards for Mathematical Content, are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect mathematical practices to mathematical content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, pause for an overview, or deviate from a known procedure to find a shortcut. Thus, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Practice and the Standards for Mathematical Content. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the necessary time, resources, innovative energies, and focus to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Through the Lens

Instructions: Use the chart below to make notes as you watch the video.

Observation Lens	Standard for Mathematical Practice that was Supported
Teacher's Questions	
Student Discussions	
Classroom Culture	

0. Setting Goals

- **It involves:**
  - Identifying what students are to know and understand about mathematics as a result of their engagement in a particular lesson
  - Being as specific as possible so as to establish a clear target for instruction that can guide the selection of instructional activities and the use of the five practices
- **It is supported by:**
  - Thinking about what students will come to know and understand rather than only on what they will do
  - Consulting resources that can help in unpacking big ideas in mathematics
  - Working in collaboration with other teachers

1. Anticipating

likely student responses to mathematical problems

- **It involves considering:**
  - The array of strategies that students might use to approach or solve a challenging mathematical task
  - How to respond to what students produce
  - Which strategies will be most useful in addressing the mathematics to be learned
- **It is supported by:**
  - Doing the problem in as many ways as possible
  - Doing so with other teachers
  - Drawing on relevant research
  - Documenting student responses year to year

2. Monitoring

students' actual responses during independent work

- **It involves:**
  - Circulating while students work on the problem and watching and listening
  - Recording interpretations, strategies, and points of confusion
  - Asking questions to get students back "on track" or to advance their understanding
- **It is supported by:**
  - anticipating student responses beforehand
  - Using recording tools

3. Selecting

student responses to feature during discussion

- **It involves:**
  - Choosing particular students to present because of the mathematics available in their responses
  - Making sure that over time all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence
  - Gaining some control over the content of the discussion (no more "who wants to present next")
- **It is supported by:**
  - Anticipating and monitoring
  - Planning in advance which types of responses to select

4. Sequencing

student responses during the discussion

- **It involves:**
  - Purposefully ordering presentations so as to make the mathematics accessible to all students
  - Building a mathematically coherent story line
- **It is supported by:**
  - Anticipating, monitoring, and selecting
  - During anticipation work, considering how possible student responses are mathematically related

5. Connecting

student responses during the discussion

- **It involves:**
  - Encouraging students to make mathematical connections between different student responses
  - Making the key mathematical ideas that are the focus of the lesson salient
- **It is supported by:**
  - Anticipating, monitoring, selecting, and sequencing
  - During planning, considering how students might be prompted to recognize mathematical relationships between responses

Anticipate Strategies That Students Might Use to Solve the Tasks and Monitor Their Work

Teaching in a manner that productively makes use of students' ideas and strategies that are generated by high-level tasks is demanding. It requires knowledge of mathematics content, knowledge of student thinking, knowledge of pedagogical "moves" that a teacher can make to lead discussions, and the ability to rapidly apply all of these in specific circumstances (M. Smith & Stein, 2011). To support teachers in this endeavor, Smith and Stein suggested five practices that are intended to make student-centered instruction more manageable. This is done by moderating the degree of improvisation required from the teacher in the midst of a discussion. Rather than providing an instant fix for mathematics instruction, the five practices provide "a reliable process that teachers can depend on to gradually improve their classroom discussions over time" (Stein, Engle, Smith, & Hughes, 2008, p. 335). The first two of the five practices are *anticipating* students' solutions to a mathematics task and *monitoring* students' actual work on the task as they work in pairs or groups. Anticipating requires considering the different ways the task might be solved. This includes anticipating factors such as how students might mathematically interpret a problem, the array of correct and incorrect strategies students might use to solve it, and how those strategies might relate to the goal of the lesson (M. Smith & Stein, 2011). Anticipating can support teachers' planning by helping them to consider, in advance, how they might respond to the work that students are likely to produce and how they can use those strategies to address the mathematics to be learned. Monitoring, as described by M. Smith and Stein (2011), is attending to the thinking of students during the actual lesson as they work either individually or collectively on the task.

This involves not only listening to students' discussions with their peers, but also observing what they are doing and keeping track of the approaches students are using. Monitoring can support teachers by allowing them to help students get ready for the classroom discussion (e.g., asking students to have an explanation prepared that uses mathematically precise language). It can also help teachers identify strategies that will advance the "collective reflection" (Cobb, Boufi, McClain, & Whitenack, 1997) of the classroom community and prepare for the end-of-class discussion (M. Smith & Stein, 2011).

Select and Sequence the Ideas to Be Shared in the Discussion

One of the primary features of a discussion-based classroom is that, instead of doing virtually all of the talking, modeling, and explaining themselves, teachers must encourage and expect students to do so. To do this effectively, teachers need to organize students' participation (National Council of Teachers of Mathematics, 1991). After monitoring the work of students as they explore the task (described above), teachers can select and sequence the ideas to be shared in the discussion (M. Smith & Stein, 2011). Selecting involves deciding which particular students will share their work with the rest of the class to get "particular pieces of the mathematics on the table" (Lampert, 2001, p. 140). Selecting which solutions will be shared by particular students is guided by the mathematical goal for the lesson and by the teacher's assessment of how each contribution will contribute to that goal. Sequencing is deciding on what order the selected students should present their work. Teachers can maximize the chances that their mathematical goals for the discussion will be achieved by making purposeful choices about the order in which students' work is shared (M. Smith & Stein, 2011). Smith and Stein suggested that teachers can also benefit from a set of moves that will help them lead whole-class discussions. Specifically, they focused on a set of "talk moves" that can be used to support students as they share their thinking with one another in respectful and academically productive ways.

Draw Connections and Summarize the Discussion

The first four of the five practices mentioned above (Anticipating, Monitoring, Selecting, and Sequencing) work to set up the discussion, whereas Connecting is primarily meant to occur during the discussion. Rather than having mathematical discussions that consist of separate presentations of different strategies and solutions, the goal is "to have student presentations build on one another to develop powerful mathematical ideas" (Smith & Stein, 2011, p. 11). The teacher supports students in drawing connections between their solutions and other solutions in the lesson. The discussion should come to an end with some kind of summary of the key mathematical ideas. The students ideally leave with "residue" from the lesson, which provides a way of talking about the understandings that remain when the activity is over (Hiebert etal., 1997).

Excerpt from NCTM Research Brief 20 (January 23, 2013) ***What Are Some Strategies For Facilitating Productive Classroom Discussions? (2-5)***

The Calling Plans Task

Company A charges a base rate of $5 per month, plus 4 cents for each minute that you're on the phone. Company B charges a base rate of only $2 per month and charges you 10 cents for every minute used. How much time per month would you have to talk on the phone before subscribing to company A would save you money?

- Solve the task in as many ways as you can, and consider other approaches that you think students might use to solve it.
- Identify errors or misconceptions that you would expect to emerge as students work on this task.