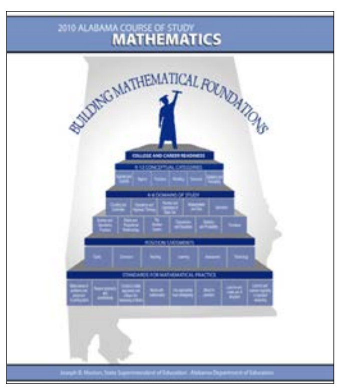
Alabama College- and Career-Ready Standards & Support

Navigating Success for All...

6 - 12

Mathematics Participant Packet



April 2014

Next Steps (to prepare for QM#4)

- Identify standards and select a high level task and plan a lesson to implement that task.
- Anticipate student responses, errors, and misconceptions. Write assessing and advancing questions related to student responses. Keep copies of planning notes.
- Teach the lesson. When you are in the Explore phase of the lesson, monitor what students are doing.
- Identify/record the approaches that can help advance the mathematical discussion later in the lesson
- **Collect student work samples and bring to the next Quarterly Meeting.**

The QM #4 goal is to be able to select, sequence and connect student work in order to orchestrate a whole-class discussion that targets the mathematical purpose(s) of the lesson.

Handout #2

Journal Reflection

How did the practice of monitoring during the “Explore Phase,” help you assess and advance students’ learning? Explain.

Thinking Through a Lesson Protocol

The main purpose of the *Thinking Through a Lesson Protocol* is to prompt you in thinking deeply about a specific lesson you will be teaching that is based on a cognitively challenging mathematical task.

SET-UP *Selecting and setting up a mathematical task*	EXPLORE *Supporting students' exploration of the task*	SHARE, DISCUSS, AND ANALYZE *Sharing and discussing the task*

▪ What are your mathematical goals for the lesson (i.e., what is it that you want students to know and understand about mathematics as a result of this lesson)?
 	
▪ In what ways does the task build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?
 	
▪ What are all the ways the task can be solved?
- Which of these methods do you think your students will use?
- What misconceptions might students have?
- What errors might students make?
 	
▪ What are your expectations for students as they work on and complete this task?
- What resources or tools will students have to use in their work?
- How will the students work – independently, in small groups, or in pairs – to explore this task?
- How long will they work individually or in small groups/pairs? Will students be partnered in a specific way? If so, in what way?
- How will students record and report their work?
 	
▪ How will you introduce students to the activity so as not to reduce the demands of the task?
 	
▪ What will you hear that lets you know students understand the task?
	▪ As students are working independently or in small groups:
	- What questions will you ask to focus their thinking?
	- What will you see or hear that lets you know how students are thinking about the mathematical ideas?
	- What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations?
	- What questions will you ask to advance students' understanding of the mathematical ideas?
	- What questions will you ask to encourage students to share their thinking with others or to assess their understanding of their peer's ideas?
	
	▪ How will you ensure that students remain engaged in the task?
	- What will you do if a student does not know how to begin to solve the task?
	- What will you do if a student finishes the task almost immediately and becomes bored or disruptive?
	- What will you do if students focus on non-mathematical aspects of the activity (e.g., spend most of their time making beautiful poster of their work)?
		▪ How will you orchestrate the class discussion so that you accomplish your mathematical goals? Specifically:
		- Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?
		- In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?
		- What specific questions will you ask so that students will:
		  ▪ make sense of the mathematical ideas that you want them to learn?
		  ▪ expand on, debate, and question the solutions being shared?
		  ▪ make connections between the different strategies that are presented?
		  ▪ look for patterns?
		  ▪ begin to form generalizations?
		
		▪ What will you see or hear that lets you know that students in the class understand the mathematical ideas that you intended for them to learn?
		
		▪ What will you do tomorrow that will build on this lesson?

Handout #4

Journal Reflection

- How did you select which students would share out? Does it really matter **who** presents a solution? Why or why not?

Select and Sequence the Ideas to Be Shared in the Discussion

One of the primary features of a discussion-based classroom is that, instead of doing virtually all of the talking, modeling, and explaining themselves, teachers must encourage and expect students to do so. To do this effectively, teachers need to organize students' participation (National Council of Teachers of Mathematics, 1991). After monitoring the work of students as they explore the task (described above), teachers can select and sequence the ideas to be shared in the discussion (M. Smith & Stein, 2011). Selecting involves deciding which particular students will share their work with the rest of the class to get "particular pieces of the mathematics on the table" (Lampert, 2001, p. 140). Selecting which solutions will be shared by particular students is guided by the mathematical goal for the lesson and by the teacher's assessment of how each contribution will contribute to that goal. Sequencing is deciding on what order the selected students should present their work. Teachers can maximize the chances that their mathematical goals for the discussion will be achieved by making purposeful choices about the order in which students' work is shared (M. Smith & Stein, 2011).

Excerpt from NCTM Research Brief 20 (January 23, 2013) What Are Some Strategies For Facilitating Productive Classroom Discussions? (4)

Handout #6

Mathematical Goals

1. recognize that there is a point of intersection between two unique nonparallel linear equations that represents where the two functions have the same x and y values.

2. understand that the two functions “switch positions” at the point of intersection and that the one that was on “top” before the point of intersection is on the “bottom” after the point of intersection because the function with the smaller rate of change will ultimately be the function closer to the x-axis.

3. make connections between tables, graphs, equations, and context by identifying the slope and y-intercept in each representational form.

The Calling Plans Task

Company A charges a base rate of $5 per month, plus 4 cents for each minute that you’re on the phone. Company B charges a base rate of only $2 per month and charges you 10 cents for every minute used. How much time per month would you have to talk on the phone before subscribing to company A would save you money?

- Solve the task in as many ways as you can, and consider other approaches that you think students might use to solve it.

- Identify errors or misconceptions that you would expect to emerge as students work on this task.

Handout #7

Anticipating Students' Responses and Monitoring Their Work

Strategy	**Who and What**	**Order**
Table	Group 1 started with increments of 1 but then gave it up and used increments of 20 Groups 2, 3, and 4 used increments of 10	
Graph	Group 1 used a calculator to create a graph from their table Group 2 made a sketch of a graph but did not plot the points Group 3 and 4 each made a graph from their table	
Equation	Group 5 made an equation and then created a graph by using 0 minutes and 100 minutes Group 6 started with the equation and used it to create a table of values incremented by 5	
Other	Group 3 had trouble understanding the context of the problem Group 4 confused the axis on their initial graph Group 6 was confused about notation and initially had used .4 instead of .04	

Fig. 4.4 Nick Bannister's completed chart for monitoring students' work on the Calling Plan task

Group 1: Tamika, Nina, Harold, Kisha

Group 2: Camilla, Jason, Lynette, Robert

Group 3: Devas, Andrea, Yolanda, Chris

Group 4: Mary, Jessica, Richard, Colin (50 minutes)

Group 5: James, Tony, Christine, Melissa

Group 6: Latasha, Derrick, Tanya, William (50 minutes)

Possible Solutions

Make a Table

Table A

Number of minutes	Cost A	Cost B
0	5.00	2.00
10	5.40	3.00
20	5.80	4.00
30	6.20	5.00
40	6.60	6.00
50	7.00	7.00
60	7.40	8.00

Same cost & min (beside the row for 50 minutes)

Because A is more than B at 40, the same as B at 50 and less than B at 60, A must become a better deal at 51 minutes.

Table B

Number of minutes	Cost A	Cost B	
0	5.00	2.00	
20	5.80	4.00	
40	6.60	6.00	A is more
60	7.40	8.00	A is less
80	8.20	10.00	
100	9.00	12.00	
120	9.80	14.00	

Because A is more than B at 40 and less than B at 60, the point of intersection must be somewhere between 40 and 60. If I graph them, I find that the point of intersection is 50. So A is a better deal starting at 51 minutes.

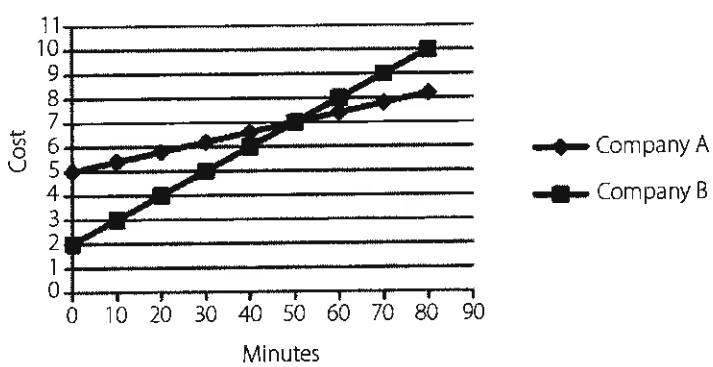
Write Equations

$y = 0.04x + 5$ (Company A)

$y = 0.10x + 2$ (Company B)

When I put the two equations into the graphing calculator, I found the point of intersection to be 50 minutes. So A becomes cheaper at 51 minutes.

Make a Graph



You can make a graph from a table of values by substituting two values for x into the equation and finding the corresponding values for y, or by putting either the table or the equation into the graphing calculator. No matter which approach you use, you find that the lines intersect at (50, 7), so plan A is better starting at 51 minutes.

Fig. 4.2. Nick Bannister's possible solutions

Journal Reflection

What are the benefits of using the "rules of thumb" as a guide when selecting and sequencing student work for the Share, Discuss, and Analyze Phase of the lesson?

Algebra I

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. (*Linear, exponential, and quadratic.*)

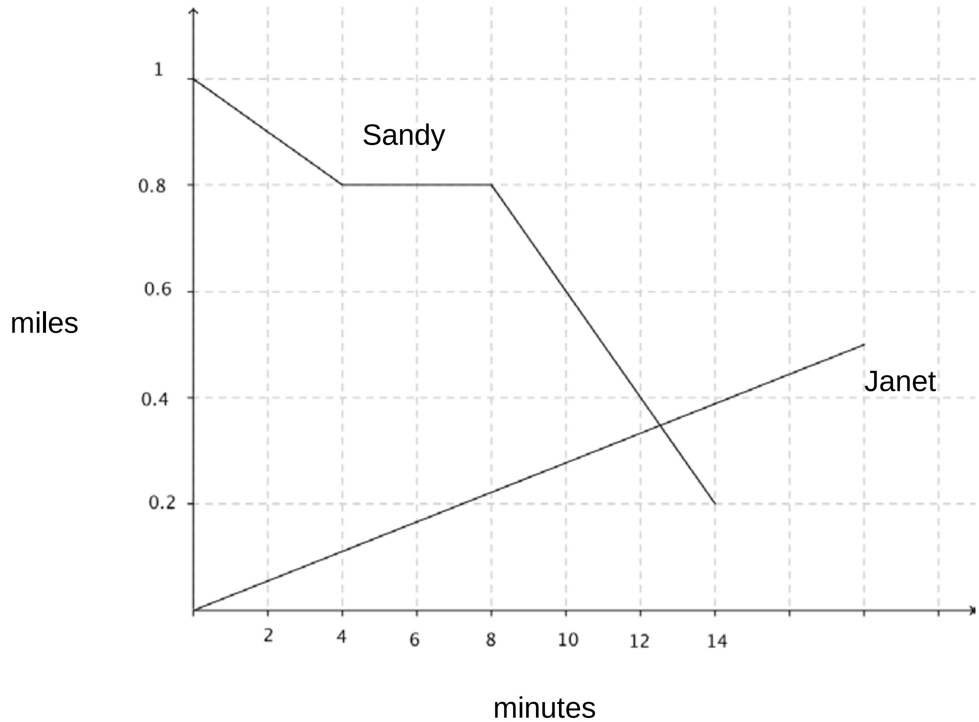
28. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** [F-IF4]

29. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Essential Understandings	CCRS
When all real numbers in an interval make sense for a contextual situation, the domain will be defined on that interval. These are functions of a continuous variable.	F-IF5
The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.	F-IF4
A rate of change describes how one variable quantity changes with respect to another – in other words, a rate of change describes the covariation between two variables (NCTM, EU 2b)	F-IF4
A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model (NCTM, EU 2c)	F-IF4 F-IF5

No Place Like Home

Two sisters, Janet and Sandy, each represented their travels from home by sketching their path on the graph shown below. The x-axis represents the time of their journey in minutes and the y-axis represents the distance from home in miles.



1. Decide whether you agree or disagree with each of the following statements. Support your answer mathematically, using specific points or time intervals where appropriate.

   a. Janet traveled mostly uphill while Sandy traveled mostly downhill.

   b. Sandy traveled at a faster rate than Janet.

   c. Sandy and Janet were at the same place at the same time once during their journeys.

   d. Each girl always traveled at a constant rate.

   e. Both girls were at home at some point during their journeys.

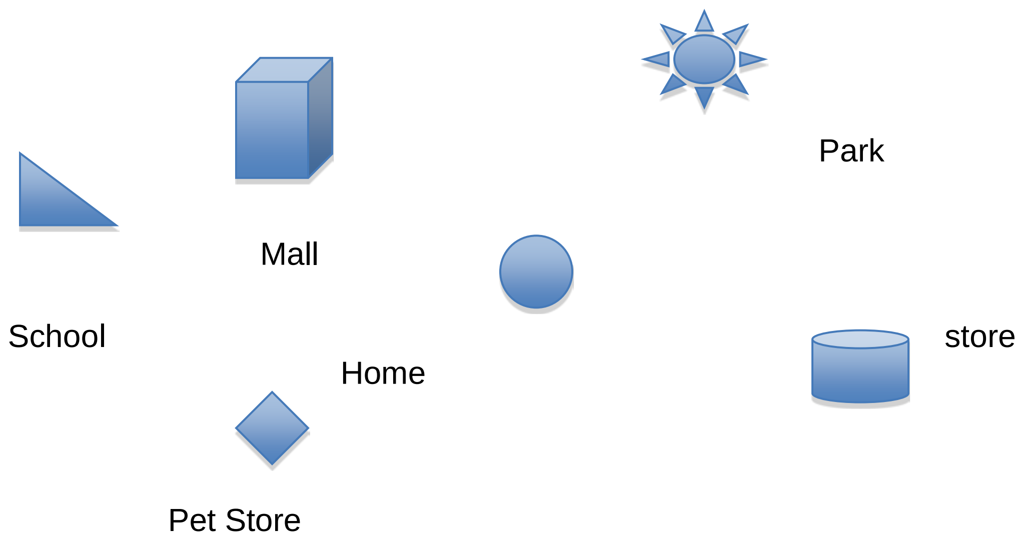
   f. Sandy stopped walking at 14 minutes.

   g. Each girl's journey represents a function.

No Place Like Home

2. Predict each of the girls' location after 22 minutes. Justify your solution mathematically.

3. Write a story for each girl's journey. You may want to use the map of their neighborhood shown below.



Extension: Assume that at 14 minutes, both girls remembered that cookies had to be taken out of the oven so they rushed home at a much faster rate than they were originally traveling. Sketch this situation on the graph and explain your reasoning.

The Task: Discussing Solution Paths

- Solve the task in as many ways as you can.
- Discuss the solution paths with colleagues at your table.
- If only one solution path has been used, work together to create others.
- Consider possible misconceptions or errors that we might see from students.

The Structure and Routines of a Lesson

Set Up of the Task

The Explore Phase/ Private Work Time
Generate Solutions

The Explore Phase/Small Group Problem Solving

1. Generate and Compare Solutions
2. Assess and Advance Student Learning

Share, Discuss, and Analyze Phase of the Lesson

1. Share and Model
2. Compare Solutions
3. Focus the Discussion on Key Mathematical Ideas
4. Engage in a Quick Write

MONITOR: Teacher selects examples for the Share, Discuss, and Analyze phase based on:

- Different solution paths to the same task
- Different representations
- Errors
- Misconceptions

SHARE: Students explain their methods, repeat others' ideas, put ideas into their own words, add on to ideas, and ask for clarification.

REPEAT THE CYCLE FOR EACH SOLUTION PATH.

COMPARE: Students discuss similarities and differences between solution paths.

FOCUS: Discuss the meaning of mathematical ideas in each representation.

REFLECT: Engage students in a Quick Write or a discussion of the process.

Analyzing Student Work

Use the student work to further your understanding of the task.

Consider:

- What do the students know?
- How did the students solve the task?
- How do their solution paths differ from each other?

Recording Sheet: Monitoring, Selecting and Sequencing Student Work

Understandings **key mathematical concepts**	**Who and What**	**Order in which the solution path will be shared**	**Rationale**

Characteristics of Questions that Support Students' Exploration

Assessing Questions

- Based closely on the work the student has produced.
- Clarify what the student has done and what the student understands about what s/he has done.
- Provide information to the teacher about what the student understands.

Advancing Questions

- Use what students have produced as a basis for making progress toward the target goal.
- Move students beyond their current thinking by pressing students to extend what they know to a new situation.
- Press students to think about something they are not currently thinking about.

Pressing for Mathematical Understanding

Let's focus on one piece of student work for the Share, Discuss, and Analyze Phase of the lesson.

Assume that a student has explained the work and others in the class have repeated the ideas and asked questions. Now it is time to "FOCUS" the discussion on an important mathematical idea.

What questions might you ask the class as a whole to focus the discussion? Write your questions below.

Group , 9

1a. Disagree. That's their distance from home not uphill/downhill.

b. Agree. Her line is going down much faster than Janet's. ← from 8 minutes on.

c. Agree. Right around 13 minutes they pass each other.

d. Disagree. Janet did her line is straight, but Sandy didn't.

e. ~~Agree~~. Disagree. Sandy stopped and didn't make it home.

f. Agree. She stopped at .2 miles

g. Disagree. Janet's is but Sandy's isn't.

2. Sandy stopped - she will be at .2. Janet will be at .7 miles. I stretched her line out.

– EU:A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model *(NCTM, EU 2c)*

Handout #17

Journal Reflection

What have you learned today that you will think about and make use of next school year? Take a few minutes and jot your thoughts down.
