THE BINOMIAL THEOREM

A quick and efficient way to expand binomials

THE BINOMIAL THEOREM

Check Answers to Worksheet

$(x+y)^0 = 1$

$(x+y)^1 = x^1 + y^1$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

What do you see???
Talk to your partner and write patterns on your worksheet.

Patterns

$(x+y)^0 = 1$

$(x+y)^1 = x^1 + y^1$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x + y)^n = x^n + \ldots + y^n$

The first term in the expansion is always the first term of the binomial to the nth power.
The last term in the expansion is always the last term of the binomial to the nth power.

Patterns

$(x+y)^0 = 1$
$(x+y)^1 = x^1 + y^1$
$(x+y)^2 = x^2 + 2xy + y^2$
$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

The sum of the exponents in any term of the expansion is equal to the exponent to which the binomial is raised.

Patterns

$(x+y)^0 = 1$
$(x+y)^1 = x^1 + y^1$
$(x+y)^2 = x^2 + 2xy + y^2$
$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

The powers of the first term in the binomial start at n and decrease to 0 through the expansion.

Patterns

$(x+y)^0 = 1$
$(x+y)^1 = x^1 + y^1$
$(x+y)^2 = x^2 + 2xy + y^2$
$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

The power of the second term in the binomial begins at 0 and increases to n through the expansion.

Patterns – What about the Coefficients?

$(x+y)^0 = 1$

$(x+y)^1 = 1x^1 + 1y^1$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Pascal's Triangle

1 Row 0
1 1 Row 1
1 2 1 Row 2
1 3 3 1 Row 3
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

The Second number in each row tells you the row number. This number should match the exponent n in the expansion $(x + y)^n$

Coefficients

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1 Row 5

The numbers in row n become the coefficients in the binomial expansion.

Return to the Worksheet and try to work #4 again.

$(x+y)^5 =$
$1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^1$

Example – You Try

Find the expansion of $(a + b)^6$ =?

$$a^6 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

What if? – Coefficients in the Binomial

If the Binomial has coefficients include them with the variable. For subtraction also include the negative with the term.

Example:

$(2x+3)^4$ =

$(2x)^4 + 4(2x)^3(3) + 6(2x)^2(3)2 + 4(2x)(3)^3 + (3)^4$

$= 16x^4 + 96x^3 + 216x^2 + 216x + 81$

You Try: $(3x – 4)^5$

$= (3x)^5 + 5(3x)^4(-4) + 10(3x)^3(-4)^2 + 10(3x)^2(-4)^3 + 5(3x)(-4)^4 + (-4)^5$

$= 243x^5 – 1620x^4 + 4320x^3 - 5760x^2 + 3240x - 1024$

What If? – Large exponent

What if the exponent is large: Do you want to write 10 or 15 rows of Pascal's Triangle every time you work a problem?

Example: $(x + 2)^{10}$ = There Must be a quicker way!

Pascal's Triangle are made up of Combinations

$_0C_0$
$_1C_0$ $_1C_1$
$_2C_0$ $_2C_1$ $_2C_2$
$_3C_0$ $_3C_1$ $_3C_2$ $_3C_3$
$_4C_0$ $_4C_1$ $_4C_2$ $_4C_3$ $_4C_4$

Instead of writing the triangle we simply use the combination formula.

$$nCr = \frac{n!}{r!\,(n-r)!}$$

This formula is programmed into your calculator.

Example: Use Combinations

$(2x - 5)^5 =$

${}_5C_0(2x)^5 + {}_5C_1(2x)^4(-5) + {}_5C_2(2x)^3(-5)^2 + {}_5C_3(2x)^2(-5)^3 + {}_5C_4(2x)^1(-5)^4 + {}_5C_5(-5)^5$
$= 1(32x^5) + 5(16x^4)(-5) + 10(8x^3)(25) + 10(4x^2)(-125) + 5(2x)(625) + 1(-3125)$
$=32x^5 - 400x^4 + 2000x^3 - 5000x^2 + 6250x - 3125$

Example: You Try

$(2x - 1)^4 =$

${}_4C_0(2x)^4 + {}_4C_1(2x)^3(-1) + {}_4C_2(2x)^2(-1)^2 + {}_4C_3(2x)^1(-1)^3 + {}_4C_4(-1)^4$

= 1(16x4) – 4(8x3) + 6(4x2) – 4(2x) +1

$= 16x^4 - 32x^3 + 24x^2 - 8x + 1$